Use 3

To establish the relation between various physical quantities (finding a formula).

If a physical quantity depends upon other physical quantities, then their relationship can be established using dimensional analysis.

Question:

The kinetic energy of a body depends upon its mass and its velocity. Find the correct physical relation of kinetic energy.

Solution: Let us denote kinetic energy, mass and velocity by E, m and v respectively.

According to question,

$E \propto m^x v^y$ *(For our convenience, we denoted K.E simply by E)*

$E = k\, m^x v^y$ ……. ….. ….. …… ……. …… (1)

Here k is a dimensionless constant.

Writing in dimensional form:

$$[E] = [m]^x [v]^y$$

Or, $$[M^1L^2T^{-2}] = [M^1L^0T^0]^x \quad [M^0L^1T^{-1}]^y$$

Or, $$[M^1L^2T^{-2}] = [M^xL^0T^0]\,[M^0L^yT^{-y}]$$

Or, $$[M^1L^2T^{-2}] = [M^{x+0}\ L^{0+y}\ \ T^{0-y}]$$

Or $$[M^1L^2T^{-2}] = [M^x\ \ L^y\ \ T^{-y}]$$

On comparing the powers of like terms, we get,

Comparing M: $x = 1$……. …… ……. (2)

Comparing L: $y = 2$ …... ……. ……. ……. (3)

Finally using equations (2) and (3) in equation (1), we get

$$E = k\, m^1 v^2$$

Or $$E = kmv^2$$

The value of dimensionless constant k can be found experimentally. Hence the relation of kinetic energy of a body with its mass and velocity is:

$$E = kmv^2$$

$$E = \frac{1}{2}\, mv^2$$

$$Here,\ k = \frac{1}{2}\ , is\ found\ experimentally.$$

✓ *Proportionality constant and dimensionless constant are different. Proportionality constant may have dimension (e.g.: G) but dimensionless constant does not have dimension (it is just a numerical value).*

Check Yourself:

1. *The centripetal force acting on a body depends upon its mass, its velocity and radius of the circle. From this information establish a formula of centripetal force.*
2. *It is known that the time period of simple pendulum depends upon the length of pendulum and acceleration due to gravity. Establish a correct relation using dimensional analysis.*