

$$\text{or } y = a \sin(\omega t - kx) + a \sin(\omega t + kx)$$

$$\text{or } y = a \left[2 \sin \left\{ \frac{\omega t - kx + \omega t + kx}{2} \right\} \cos \left\{ \frac{\omega t - kx - \omega t - k}{2} \right\} \right]$$

$$\text{or } y = 2a \sin \omega t \cos kx$$

$$\text{or } y = 2a \cos kx \sin \omega t$$

$$\text{or } y = A \sin \omega t \dots \dots \dots (3) \quad [\text{we replaced } 2a \cos kx \text{ by } A]$$

Equation (3) is the equation of stationary wave. The amplitude of the wave is: $A = 2a \cos kx$.

Since the amplitude is sinusoidal in x (position), it is maximum at some point and is minimum (zero) at some point. The points corresponding to maximum amplitude are called as **antinodes** and the points corresponding to minimum (zero) amplitude are called as **nodes**.

▪ **Positions of antinodes (maximum amplitude):**

The amplitude of stationary wave is:

$$A = 2a \cos kx$$

The amplitude will be maximum if $\cos kx$ is maximum,

$$\text{i.e., } \cos kx = \pm 1 \quad \text{and } A_{\max} = 2a$$

$$\text{or } \cos \frac{2\pi}{\lambda} x = \cos(0, \pi, 2\pi, 3\pi, \dots)$$

$$\text{or } \frac{2\pi}{\lambda} x = n\pi \quad ; n = 0, 1, 2, \dots$$

$$\text{or } x = n \frac{\lambda}{2}$$

This indicates the position of antinodes.

Distance between two successive antinodes $\Delta x = \frac{\lambda}{2}$

▪ **Position of nodes (minimum- zero amplitude)**

The amplitude will be minimum if $\cos kx$ is minimum,

$$\text{i.e., } \cos kx = 0 \quad \text{and } A_{\min} = 0$$

$$\text{or } \cos \frac{2\pi}{\lambda} x = \cos\left(\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots\right)$$

$$\text{or } \frac{2\pi}{\lambda} x = (2n + 1) \frac{\pi}{2} \quad ; n = 0, 1, 2, \dots$$

$$\text{or } x = (2n + 1) \frac{\lambda}{4}$$

This indicates the position of nodes.

Distance between two successive nodes $\Delta x = \frac{\lambda}{2}$

Distance between two successive nodes and antinodes = $\frac{\lambda}{4}$

Positions of antinodes:

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda, \dots$$

Distance between two successive antinodes $\Delta x = \frac{\lambda}{2}$

Antinode is a point of maximum displacement (or amplitude) and minimum pressure (minimum pressure variation). Hence the displacement antinode is called as pressure node.

No (minimum) sound is heard at antinode (as sound wave is a pressure wave).

Positions of nodes:

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots$$

Distance between two successive nodes $\Delta x = \frac{\lambda}{2}$

Node is a point of minimum displacement (or amplitude) and maximum pressure (maximum pressure variation). Hence the displacement node is called as pressure antinode.

Maximum (intense) sound is heard at node (as sound wave is a pressure wave).