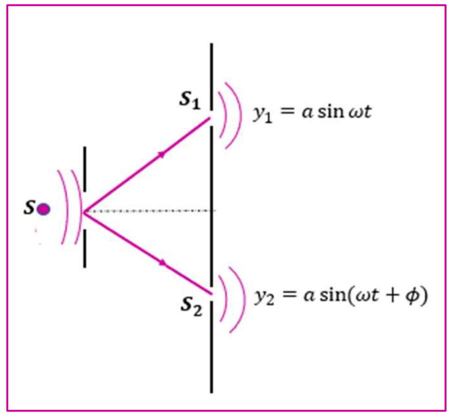
ANALYTICAL TREATMENT IF INTERFERENCE OF LIGHT

Consider two coherent light waves (from S_1 & S_2) produced from a single monochromatic source (S), as shown in figure.



The displacement equation of two waves is:

$$\boldsymbol{y_1 = a\sin\omega t}$$

and, $$\boldsymbol{y_2 = a\sin(\omega t + \phi)}$$

Here, *ϕ is the phase difference between the waves.*

From principle of superposition of waves, the displacement of resultant wave ($\boldsymbol{y}$) is:

$$y = y_1 + y_2$$

or $$y = a\sin\omega t + a\sin(\omega t + \phi)$$

or $$y = a[\sin\omega t + \sin(\omega t + \phi)]$$

or $$y = a\left[2\sin\left(\frac{\omega t + \omega t}{2}\right)\ \cos\left(\frac{\omega t - \omega t - \phi}{2}\right)\right]$$

or $$\boxed{y = 2a\cos\left(\frac{\phi}{2}\right)\ \sin\ \left(\omega t + \frac{\phi}{2}\right)}$$ **: Equation of resultant wave.**

Here, $2a\cos\left(\frac{\phi}{2}\right)$ is the amplitude of resultant wave.

i. Conditions and positions of maxima (constructive interference):

The intensity of the resultant wave is maximum if amplitude $\boldsymbol{2a\cos\left(\frac{\phi}{2}\right)}$ is maximum.

i.e., $$\cos\left(\frac{\phi}{2}\right) = \pm 1$$

or $$\cos\left(\frac{\phi}{2}\right) = \cos n\pi \qquad (n = 0, 1, 2, 3 \ldots)$$

or $$\frac{\phi}{2} = n\pi$$

$$\boxed{\phi = 2n\pi}$$: phase difference for constructive interference.

With, $\quad A_{max} = \pm 2a$

$\because I \propto A^2 \qquad \therefore\ I = 4a^2\cos^2\frac{\phi}{2}$

And $\quad I_{max} = 4a^2$

For constructive interference, the **phase difference** between two interfering waves should be **even multiple of π.**

or $$\frac{1}{2}\frac{2\pi}{\lambda}x = n\pi \quad [\because (phase\ difference), \phi = \frac{2\pi}{\lambda} \times x\ (path\ difference)]$$

or $$\boxed{x = n\lambda}$$: path difference for constructive interference.

For constructive interference, the **path difference** between two interfering waves should be **integral multiple of λ.**

ii. Conditions and positions of minima (destructive interference):

The intensity of the resultant wave is minimum if amplitude $\boldsymbol{2a\cos\left(\frac{\phi}{2}\right)}$ is minimum.

i.e., $$\cos\left(\frac{\phi}{2}\right) = 0$$

or $$\cos\left(\frac{\phi}{2}\right) = \cos\,(2n-1)\frac{\pi}{2} \qquad (n = 1, 2, 3 \ldots)$$

or $$\frac{\phi}{2} = (2n-1)\frac{\pi}{2}$$

$$\boxed{\phi = (2n-1)\pi}$$: phase difference for destructive interference.

With, $\quad A_{min} = 0$

And $\quad I_{min} = 0$

For destructive interference, the **phase difference** between two interfering waves should be **odd multiple of π.**

or $$\frac{1}{2}\frac{2\pi}{\lambda}x = (2n-1)\frac{\pi}{2}$$