$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{a^2}$$
$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin 90}{a^2}$$
$$dB = \frac{\mu_0}{4\pi}\frac{Idl}{a^2} - - - - - - - - (1)$$

Now, Total magnetic field due to the circular coil can be obtained by integrating equation (1) from limit 0 to $2\pi a$.

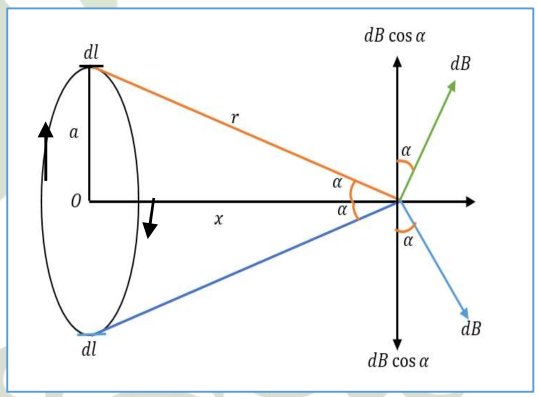
i.e.
$$B = \int_0^{2\pi a} dB = \int_0^{2\pi a} \frac{\mu_0}{4\pi}\frac{Idl}{a^2}$$
$$B = \frac{\mu_0}{4\pi}\frac{I}{a^2}\int_0^{2\pi a} dl$$
$$B = \frac{\mu_0}{4\pi}\frac{I}{a^2}2\pi a$$
$$B = \frac{\mu_0 I}{2a}$$

For N number of turns,
$$B = \frac{\mu_0 NI}{2a}$$

#NP: A wire of length 62.8m Carrying current 10A is bent into a circular coil of radius 10cm. find magnetic field at center. (Ans: 6.28×10^{-3}T)

#NP: An electron is revolving around its orbit of radius 5×10^{-11}m with a frequency of 2×10^{10} Hz. Calculate magnitude of magnetic field at its center. (Ans: 4.02×10^{-5}T)

2. Magnetic Field on the axis of the coil:



Let us take on narrow circular coil of radius $'a'$ centered at $'O'$ and carrying current $'I'$ in clockwise direction. We have to calculate magnetic field at point along x-axis at a distance $'x'$ from center $'O'$. Let us take small element of length $'dl'$ at point in circumference. The angle between dl and r is 90^0.

According to Biot & Savart law, magnetic field at point (P) due to the small element (dl),

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$
$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin 90}{r^2}$$
$$dB = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}$$ (Direction along P to outward)

On resolving dB into its constituent components, we get $dB\sin\theta$ and $dB\cos\theta$ as shown in figure.

Let us take another small element (dl') at diametrically opposite (dl) since all the components $dB\cos\theta$ being equal in magnitude, acting in opposite direction cancel to each other. However, all the components $dB\sin\theta$ are being equal in magnitude and acting in same direction are added up. Now, total magnetic field can be obtained by integrating either $dB\sin\theta$ from limit 0 to $2\pi a$.

i.e.
$$B = \int_0^{2\pi} dB\sin\theta = \int_0^{2\pi a}\frac{\mu_0}{4\pi}\frac{Idl}{r^2}\sin\theta$$
$$B = \frac{\mu_0}{4\pi}\frac{I}{r^2}\int_0^{2\pi a}\frac{a}{r}dl \quad [\sin\theta = \frac{a}{r}]$$
$$B = \frac{\mu_0}{4\pi}\frac{Ia}{r^3}\int_0^{2\pi a} dl$$