$$
\begin{aligned}
B & =\frac{\mu_{0}}{4 \pi} \frac{I a}{r^{3}} 2 \pi a \\
B & =\frac{\mu_{0}}{2} \frac{I a^{2}}{r^{3}} \\
B & =\frac{\mu_{0} I a^{2}}{2\left(a^{2}+x^{2}\right)^{3 / 2}} \quad\left[r=\sqrt{a^{2}+x^{2}}\right]
\end{aligned}
$$

For N turns,

$$
B=\frac{\mu_{0} N I^{2}}{2\left(a^{2}+x^{2}\right)^{3 / 2}} \quad\left[r=\sqrt{a^{2}+x^{2}}\right]
$$

Special Cases:

1. If, $x \ggg a$, in this case the value of a can be neglected as compare to $x$,

$$
B=\frac{\mu_{0} N I a^{2}}{2\left(x^{2}\right)^{3 / 2}}=\frac{\mu_{0} N I a^{2}}{2 x^{3}}
$$

2. If $x=0$,

$$
B=\frac{\mu_{0} N I a^{2}}{2 a^{3}}=\frac{\mu_{0} N I}{2 a}
$$

This is the case of magnetic field due to circular coil at the center.

## Note:

- If the point on the axis of the coil is at a distance equal to radius of the coil,

$$
\text { i.e. } x=a \text {, then, } \quad B=\frac{\mu_{0} N I}{\sqrt{32} a}
$$

- The variation of magnetic field $(B)$ and distance $(r)$ of a point on the axis of circular coil carrying current is represented as:


## Magnetic Field due to straight conductor:

Let us take a straight conductor carrying current ' $I$ ' in upward direction. We have to calculate magnetic field at point ${ }^{\prime} P^{\prime}$ at a distance ' $a a^{\prime}$ from the straight conductor i.e. $O P=a$. Let us take small element of length ' $d l^{\prime}$. According to Biot \& Savart law, magnetic field at point ' $P$ ' due to the small element ${ }^{\prime} d l^{\prime}$,

$$
d B=\frac{\mu_{0}}{4 \pi} \frac{I d l \sin (90-)}{r^{2}}--(1)
$$

From fig,

$$
\begin{aligned}
& \sin (90-\theta)=\frac{a}{r}=\cos \theta \\
& r=a \sec \theta---(2) \\
& \tan \theta=\frac{l}{a} \\
& l=a \tan \theta \\
& d l=\operatorname{asec}^{2} \theta d \theta--(3)
\end{aligned}
$$

And,

Using eq. (2) and (3) in eq. (1),

$$
\begin{aligned}
d B & =\frac{\mu_{0}}{4 \pi} \frac{I d l \cos \theta}{r^{2}} \\
d B & =\frac{\mu_{0}}{4 \pi} \frac{I \sec c^{2} \theta d \theta \cos \theta}{(a \sec )^{2}} \\
d B & =\frac{\mu_{0} I}{4 \pi a} \cos \theta d \theta---(4)
\end{aligned}
$$

Now, total magnetic field can be obtained by integrating equation (4) from limit $-\alpha_{1}$ to $\alpha_{2}$

$$
B=\int_{-\alpha_{1}}^{\alpha_{2}} d B
$$

