

$$\text{or, } -\frac{dx}{d\alpha} = a(-\text{cosec}^2\alpha)$$

$$\text{or, } dx = a\text{cosec}^2\alpha d\alpha \text{ ----- (3)}$$

Using equation (2) and (3) in (1), we get

$$\text{or, } dB = \frac{\mu_0 n I a^2 a \text{cosec}^2\alpha d\alpha}{2(a^2 + (a \cot \alpha)^2)^{3/2}}$$

$$\text{or, } dB = \frac{\mu_0 n I a^2 a \text{cosec}^2\alpha d\alpha}{2a^3 \text{cosec}^3\alpha}$$

$$\text{or, } dB = \frac{\mu_0 n I}{2} \sin\alpha d\alpha \text{ ----- (4)}$$

Now, the magnetic field at 'P' due to whole length of the solenoid is obtained by integrating equation (4) between the limits  $\alpha_1$  to  $\alpha_2$ .

i.e.

$$B = \int_{\alpha_1}^{\alpha_2} \frac{\mu_0 n I}{2} \sin\alpha d\alpha$$

$$\text{or, } B = \frac{\mu_0 n I}{2} \int_{\alpha_1}^{\alpha_2} \sin\alpha d\alpha$$

$$\text{or, } B = \frac{\mu_0 n I}{2} [-\cos\alpha]_{\alpha_1}^{\alpha_2}$$

$$B = \frac{\mu_0 n I}{2} [\cos\alpha_1 - \cos\alpha_2]$$

For infinitely long solenoid,  $\alpha_1 = 0^\circ$  and  $\alpha_2 = 180^\circ$

$$\text{or, } B = \frac{\mu_0 n I}{2} [\cos 0^\circ - \cos 180^\circ]$$

$$B = \mu_0 n I$$

Note:

- A solenoid consists of an insulated long wire closely wound in the form of a helix.
- Its length is very large as compared to its diameter.
- If the solenoid is sufficiently long, the field within it is uniform (except at the ends).
- The magnetic field lines due to current carrying solenoid resemble exactly with those of a bar magnet.
- The magnetic field at the ends of a very long current carrying solenoid is half of that at the center.
- The magnetic field outside the solenoid is negligible (zero).

### Ampere's Circuital Law:

- This is the alternative method to Biot-Savart's law to calculate magnitude of magnetic field.
- This law is more convenient while calculating magnetic fields of current distribution with a high degree of symmetry. It is more useful if direction of field is tangential to length element.

Statement: It states that the line integral of magnetic field intensity over a closed path in free space is equal to  $\mu_0$  times the net current enclosed by the closed path. i.e.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{en}$$

Where,  $I_{en}$  = net current enclosed, and  $\mu_0$  is permeability of free space.