## Torque on a rectangular coil in a uniform field:

Principle: When an electric current flow in a rectangular coil of wire placed in a magnetic field, the magnetic forces produce a torque, which tends to rotate the coil about its axis.
Application: Galvanometer, Loudspeaker
Let a rectangular coil PQRS of length $\mathrm{PQ}=\mathrm{RS}=1$ and breadth $\mathrm{QR}=\mathrm{SP}=$ b , carrying current. I in anticlockwise direction is suspended in uniform magnetic field B. Suppose at an instant of time the plane of coil makes an angle $\theta$ with the direction of magnetic field i.e. the breadth wise position of the coil makes an angle $\theta$ with magnetic field but length wise position is always perpendicular to the direction of magnetic field. When current flows through the rectangular coil, it's each arm experience force. The force experienced by each arm is given below.


Fig. Torque on coil at an angle $\theta$ with B

Force on PQ,

$$
\begin{align*}
& \overrightarrow{F_{4}}=I(\vec{l} X \vec{B})[P Q=l] \\
& F_{4}=I l B \sin \theta \\
& F_{4}=I l B[\theta=90] \ldots \ldots \tag{1}
\end{align*}
$$

Force on arm QR

$$
\begin{align*}
& \overrightarrow{F_{1}}=I(\vec{b} X \vec{B})[Q R=b] \\
& F_{1}=I b B \sin \theta \ldots \ldots \ldots \ldots \tag{2}
\end{align*}
$$

Force on arm RS

$$
\begin{align*}
& \overrightarrow{F_{3}}=I(\vec{l} X \vec{B})[R S=l] \\
& F_{3}=I l B \sin \theta \\
& F_{3}=I l B[\theta=90] \ldots . . \tag{3}
\end{align*}
$$


fig: Top view of rectangular coil

Force on arm SP

$$
\begin{align*}
& \overrightarrow{F_{2}}=I(\vec{b} X \vec{B})[S P=b] \\
& F_{2}=I b B \sin (180-\theta) \\
& F_{2}=I b B \sin \theta \ldots \ldots \ldots \ldots \tag{4}
\end{align*}
$$

The force $F_{1}$ an $F_{2}$ being equal in magnitude acting in opposite direction with same line of action cancel to each other but the forces $F_{3}$ and $F_{4}$ being equal in magnitude acting in opposite direction with different line of action constitute a couple. The torque due to couple of forces $F_{3}$ and $F_{4}$ is,
Torque $(\tau)=$ Force on either arm $\times$ Perpendicular distance
Or, $\tau=F_{4} \times \mathrm{NS}$
Or, $\tau=F_{4} \mathrm{PS} \cos \theta[\because$ from $\Delta \mathrm{PNS} \cos \theta=\mathrm{NS} / \mathrm{PS}]$
Or, $\tau=$ BI l b $\cos \theta[\because \mathrm{PS}=\mathrm{b}]$
Or, $\tau=$ BIA $\cos \theta$
For N -turns,
$\tau=$ BINA $\boldsymbol{\operatorname { c o s }} \theta$
(5)

In addition, if we take the angle between magnetic field and normal to the plane of loop then eqn (5) can be written as, $\alpha=90-\theta$

$$
\begin{equation*}
\tau=\text { BINA } \sin \alpha- \tag{6}
\end{equation*}
$$

Special cases

