Half-life:

The half-life of a radioactive element (substance) is defined as the time interval during which the number of atoms decreases to half of the initial number.

If $T_{1/2}$ is the half-life period of a radioactive substance then

For, $t = T_{1/2}$, $N = \frac{N_o}{2}$

Since, $N = N_o\, e^{-\lambda}$

$$\therefore \quad \frac{N_o}{2} = N_o\, e^{-\lambda T_{1/2}}$$

$$or, \quad \frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$or, \quad 2 = e^{\lambda T_{1/2}}$$

Taking ln on both sides, we get

$$\ln 2 = \lambda\, T_{1/2}$$

$$or, \quad T_{1/2} = \frac{ln\, 2}{\lambda}$$

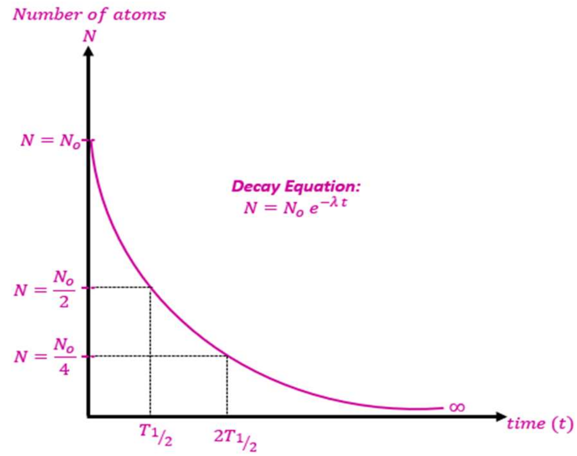

Figure: Decay Curve

$$\therefore \quad \boldsymbol{T_{1/2} = \frac{0.693}{\lambda}}$$

: Relation between half-life and decay constant.

Mean-Life ($\bar{T}$):

The mean life of a radioactive substance is defined as the ratio of sum of lives of all radioactive atoms to total number of radioactive atoms.

i.e., $$\boldsymbol{mean\ life = \frac{\text{sum of lives of all radioactive atoms}}{\text{total number of radioactive atoms}}}$$

Mathematically,

✓ $mean\ life = \frac{1}{\lambda}$

✓ $mean\ life = \sqrt{2}\, T_{1/2}$

Radioactive decay constant: [Definition of λ]

Since,

$$\boldsymbol{\frac{dN}{dt} = \lambda N} \qquad \boldsymbol{(magnitude\ only)}$$

$$\therefore \boldsymbol{\lambda = \frac{dN/dt}{N}}$$

Thus, decay constant of a radioactive substance is defined as the ratio of the rate of disintegration of radioactive atoms at a given instant of time to the number of atoms present at that instant.

Also, N = N₀ $\boldsymbol{e^{-\lambda t}}$

If, $\boldsymbol{\lambda = \frac{1}{t}}$**, (reciprocal of time)**

Then, $\boldsymbol{N = N_o\, e^{-\frac{1}{t} \times t}}$

$$\boldsymbol{N = N_o e^{-1}}$$

$$\boldsymbol{N = 0.368\, N_o}$$

$$\boldsymbol{N = 37\%\ of\ N_o}$$

OR,
Number of radioactive substances decreases by about 63% of their original number.

Hence the radioactive decay constant may also be defined as the reciprocal of time in which the number of atoms of a radioactive substance decreases to about 37% of their original (initial) number.