ERROR ANALYSIS

The process of evaluating the uncertainty associated with a measurement result is often called uncertainty analysis or error analysis.

#1: Absolute error

The difference between the actual value (true value) and the measured value is called absolute error.

Absolute error = |true value - observed value|

e.g., let a quantity is measured 'n' times and observed values are $x_1, x_2, x_3, x_4, \dots, x_n$,

(if true value/standard value is unknown) Then their true value is calculated as arithmetic mean of observed values, i.e. $\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 \dots + x_n}{n}$ Now the absolute error is given as,

$$\begin{array}{l} \Delta x_1 = \bar{x} - x_1 \\ \Delta x_2 = \bar{x} - x_2 \\ \Delta x_n = \bar{x} - x_n \end{array}$$

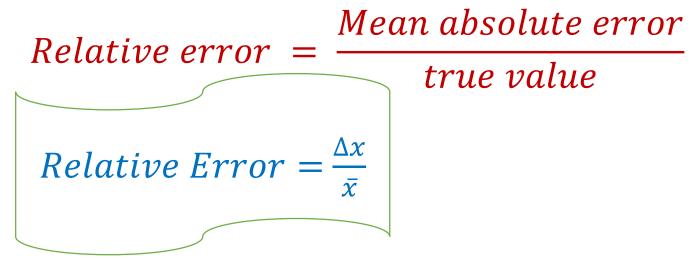
Mean absolute error:

$$\Delta x = \frac{\Delta x_2 + \Delta x_2 + \Delta x_3 + \dots + \Delta x_n}{n}$$

And the measured value is written as, Measured value = $(\bar{x} \pm \Delta x)$

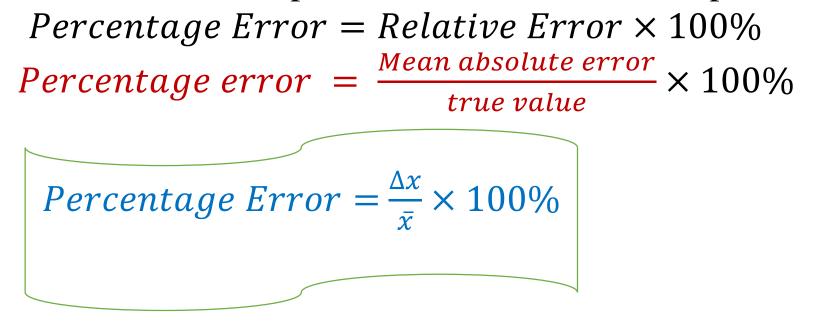
#2: Relative Error

The ratio of the mean absolute error and the true value is called the relative error.



#3: Percentage Error:

When the relative error is expressed as % it is called the percentage error.



1. Addition & Subtraction

When physical quantities are added or subtracted then the maximum error in the result is the sum of the errors of the individual quantities. In Addition:

> Let the sum of two quantities be: Maximum error in measurement is: result:

$$x = a + b$$

$$\Delta x = \Delta a + \Delta b$$

$$\Rightarrow (x \pm \Delta x)$$

Then,

e.g.

The least count is 0.01mm. Two wires of length L_1 and L_2 are measured and they are connected forming a single wire. Then the measurement is,

A. $(L_1 + L_2)m \pm 0.02mm$ B. $(L_1 - L_2)m \pm 0.02mm$ C. $(L_1 + L_2)m \pm 0.01mm$ D. $(L_1 - L_2)m \pm 0.01mm$

1. Addition & Subtraction

When physical quantities are added or subtracted then the maximum error in the result is the sum of the errors of the individual quantities. In Addition:

Let the sum of two quantities be: x = a + bMaximum error in measurement is: $\Delta x = \Delta a + \Delta b$ $\Rightarrow (x \pm \Delta x)$ result: In Subtraction:

Let the difference of two quantities be: x = a - bMaximum error in measurement is: $\Delta x = \Delta a + \Delta b$ result: $\Rightarrow (x \pm \Delta x)$

Then,

Then,

2. Multiplication & Division

In multiplication and division the error in the result is the sum relative error of respective quantities used in calculation.

In multiplication:

Then,

Let the multiplication of two quantities be: x = aberror in measurement is: $\frac{\Delta x}{x} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$ $\Delta x = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) x$ result: $\Rightarrow (x \pm \Delta x)$ $\frac{\Delta a}{\Delta a}$ =relative error in the measurement of a a $\frac{\Delta b}{b}$ =relative error in the measurement of b E.g. The length and breadth of rectangle is found to be (6.2 ± 0.2) cm & (2.3 ± 0.1) cm calculate the error in area.

In Division:

Let the division of two quantities be: $x = \frac{a}{b}$ error in measurement is: $\frac{\Delta x}{x} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$ $\Delta x = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) x$ Then, result: $\Rightarrow (x \pm \Delta x)$

e.g. The distance travelled by the particle in time $(2.5 \pm 0.1)s$ is $(12.5 \pm 0.3)m$, calculate the error in velocity.

3. Mixed quantities having power:

Suppose a quantity x depends upon the quantities a, b & c according to the equation, $a^{p}b^{q}$

Where, p, q & r are numbers. Then the relative error in the measurement of x is given by

$$\frac{\Delta x}{x} = p\left(\frac{\Delta a}{a}\right) + q\left(\frac{\Delta b}{b}\right) + r\left(\frac{\Delta c}{c}\right)$$

Similarly in terms of percentage error:

$$\frac{\Delta x}{x} \times 100\% = p\left(\frac{\Delta a}{a} \times 100\%\right) + q\left(\frac{\Delta b}{b} \times 100\%\right) + r\left(\frac{\Delta c}{c} \times 100\%\right)$$

Where,

$$\frac{\Delta a}{\frac{a}{b}} = relative \ error \ in \ the \ measurement \ of \ a$$
$$\frac{\frac{\Delta b}{b}}{\frac{b}{c}} = relative \ error \ in \ the \ measurement \ of \ b$$
$$\frac{\Delta c}{c} = relative \ error \ in \ the \ measurement \ of \ c$$

e.g.

- 1. The percentage error in measurement of mass and speed are 2% and speed are 3% respectively. What will be the error in the measurement of kinetic energy?
- 2. The error in measurement of radius of the sphere is 2%, then what will be the possible error in measurement of volume?
- 3. If the change in KE is 4%, then momentum changes by