ERROR ANALYSIS

The process of evaluating the uncertainty associated with a measurement result is often called uncertainty analysis or error analysis.

#1: Absolute error

The difference between the actual value (true value) and the measured value is called absolute error.

$$Absolute\ error\ = |true\ value - observed\ value|$$

e.g., let a quantity is measured $'n'$ times and observed values are $x_1, x_2, x_3, x_4, \ldots\ldots\ldots x_n,$

(if true value/standard value is unknown)

Then their true value is calculated as arithmetic mean of observed values,

i.e. $$\bar{x} = \frac{x_1+x_2+x_3+x_4 \ldots +x_n}{n}$$

Now the absolute error is given as,

$$\Delta x_1 = \bar{x} - x_1$$
$$\Delta x_2 = \bar{x} - x_2$$
$$\Delta x_n = \bar{x} - x_n$$

Mean absolute error:

$$\Delta x = \frac{\Delta x_2 + \Delta x_2 + \Delta x_3 + \cdots + \Delta x_n}{n}$$

And the measured value is written as,

$$\text{Measured value} = (\bar{x} \pm \Delta x)$$

#2: Relative Error

The ratio of the mean absolute error and the true value is called the relative error.

$$Relative\ error = \frac{Mean\ absolute\ error}{true\ value}$$

$$Relative\ Error = \frac{\Delta x}{\bar{x}}$$

#3: Percentage Error:

When the relative error is expressed as % it is called the percentage error.

$$Percentage\ Error = Relative\ Error \times 100\%$$

$$Percentage\ error\ = \frac{Mean\ absolute\ error}{true\ value} \times 100\%$$

$$Percentage\ Error = \frac{\Delta x}{\bar{x}} \times 100\%$$

Propagation of Errors:

1. Addition & Subtraction

When physical quantities are added or subtracted then the maximum error in the result is the sum of the errors of the individual quantities.

In Addition:

Let the sum of two quantities be: $x = a + b$

Maximum error in measurement is: $\Delta x = \Delta a + \Delta b$

Then, result: $\Rightarrow (x \pm \Delta x)$

e.g.

The least count is 0.01mm. Two wires of length L_1 and L_2 are measured and they are connected forming a single wire. Then the measurement is,

A. $(L_1 + L_2)m \pm 0.02mm$

B. $(L_1 - L_2)m \pm 0.02mm$

C. $(L_1 + L_2)m \pm 0.01mm$

D. $(L_1 - L_2)m \pm 0.01mm$

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In Addition:

Let the sum of two quantities be: $x = a + b$

Maximum error in measurement is: $\Delta x = \Delta a + \Delta b$

Then, result: $\Rightarrow (x \pm \Delta x)$

In Subtraction:

Let the difference of two quantities be: $x = a - b$

Maximum error in measurement is: $\Delta x = \Delta a + \Delta b$

Then, result: $\Rightarrow (x \pm \Delta x)$

Propagation of Errors:

2. Multiplication & Division

In multiplication and division the error in the result is the sum relative error of respective quantities used in calculation.

In multiplication:

Let the multiplication of two quantities be: $x = ab$

error in measurement is: $\frac{\Delta x}{x} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$

$$\Delta x = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) x$$

Then, result: $\Rightarrow (x \pm \Delta x)$

$\frac{\Delta a}{a}$ =relative error in the measurement of a

$\frac{\Delta b}{b}$ =relative error in the measurement of b

E.g. The length and breadth of rectangle is found to be (6.2 ± 0.2)cm & $(2.3 \pm 0.1)cm$ calculate the error in area.

Propagation of Errors:

In Division:

Let the division of two quantities be: $x = \frac{a}{b}$

error in measurement is: $\frac{\Delta x}{x} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$

$$\Delta x = \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right) x$$

Then, result: $\Rightarrow (x \pm \Delta x)$

e.g. The distance travelled by the particle in time $(2.5 \pm 0.1)s$ is $(12.5 \pm 0.3)m$, calculate the error in velocity.

3. Mixed quantities having power:

Suppose a quantity x depends upon the quantities a, b & c according to the equation,

$$x = \frac{a^p b^q}{c^r}$$

Where, p, q & r are numbers. Then the relative error in the measurement of x is given by

$$\frac{\Delta x}{x} = p\left(\frac{\Delta a}{a}\right) + q\left(\frac{\Delta b}{b}\right) + r\left(\frac{\Delta c}{c}\right)$$

Similarly in terms of percentage error:

$$\frac{\Delta x}{x} \times 100\% = p\left(\frac{\Delta a}{a} \times 100\%\right) + q\left(\frac{\Delta b}{b} \times 100\%\right) + r\left(\frac{\Delta c}{c} \times 100\%\right)$$

Where,

$$\frac{\Delta a}{a} = relative\ error\ in\ the\ measurement\ of\ a$$

$$\frac{\Delta b}{b} = relative\ error\ in\ the\ measurement\ of\ b$$

$$\frac{\Delta c}{c} = relative\ error\ in\ the\ measurement\ of\ c$$

e.g.

1. The percentage error in measurement of mass and speed are 2% and speed are 3% respectively. What will be the error in the measurement of kinetic energy?
2. The error in measurement of radius of the sphere is 2%, then what will be the possible error in measurement of volume?

3. If the change in KE is 4%, then momentum changes by